

科目：工程數學(線性代數、微分方程)

適用：電機系

編號：342

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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線性代數，50%

1. (15%) Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 4 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{bmatrix}.$$

Find the following:

- (a) (5%) $\text{rank}(A)$,
 - (b) (5%) $\text{nullity}(A)$,
 - (c) (5%) An orthogonal basis for the column space of A .
2. (20%) True or false: (A and B are $n \times n$ matrices)
- (a) (5%) If $\text{rank}(A) < n$, then $\text{rank}(AB) < n$.
 - (b) (5%) $\text{nullity}(A + B) = \text{nullity}(A) + \text{nullity}(B)$.
 - (c) (5%) If A is orthogonal ($A^T A = I$), then $\text{tr}(A) = n$.
 - (d) (5%) For any A, B , $\det(AB) = \det(A) \det(B)$.
3. (15%) True or false:
- (a) (3%) If A has eigenvalue 0, then A^T also has eigenvalue 0.
 - (b) (3%) Similar matrices A and B always share the same eigenvectors.
 - (c) (3%) If λ is non-zero and is an eigenvalue of AB , then λ is also an eigenvalue of BA . (assuming AB, BA exist)
 - (d) (3%) The zero matrix has no eigenvalues.
 - (e) (3%) If v is an eigenvector of A with eigenvalue λ , then $v^T A = \lambda v^T$.

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4. Consider an ODE $(\cos y + e^{-x})y' + \sin y = 2xe^{-x}$.

(a) Is the ODE an exact ODE? Please explain your answer. (5%)

(b) Find the general solution of the ODE. (10%)

[提示：(a)需要說明答案]

5. Consider an ODE $(x^2 + 3x + 2)y'' - (x + 2)y' + y = 0$.

(a) One solution is $(x + \alpha)$. Find the value of α . (5%)

(b) Solve the ODE by the method of reduction of order (降階法). (15%)

6. (a) One solution of $y'' + 2y' - 3y = \beta e^{2x}$ is e^{2x} . Find β . (5%)

(b) Find the general solution of $y'' + 2y' - 3y = 4e^x + \beta e^{2x}$. (10%)